



MATHEMATICS ASSIGNMENT

Class: 12 A, B, C

Time: 4 hours

General Instructions:

- This assignment gives you points for Recall and questions from each of the three topics of
Relations and functions
Matrices and determinants
Continuity and differentiability
- The assignment is to be done in your Question Bank Register.
- Take your time for recall and memorizing the formulae and results. The questions should take you no more than four hours.
- The assignment is to be submitted on the 8th of July, it will be graded and the grades will go towards grades for the month of July.

RELATIONS AND FUNCTIONS

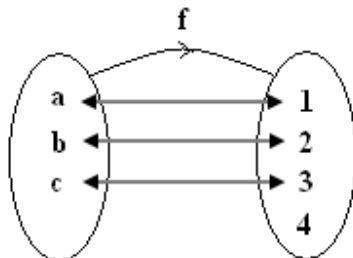
Equivalence Relation:

Reflexive Relation: A relation R in the set A is called reflexive relation if $(a, a) \in R$, for every $a \in A$.

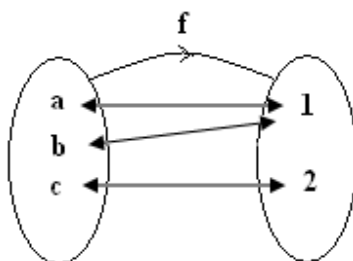
Symmetric Relation: A relation R in the set A is called symmetric, if $(a, b) \in R$ implies that $(b, a) \in R$, for all $a, b \in A$.

Transitive Relation: A relation R in the set A is called transitive, if $(a, b) \in R$ and $(b, c) \in R$ implies that $(a, c) \in R$, for all $a, b, c \in A$.

One-one Function: A function $f : A \rightarrow B$ is one-one or injective, if $f(a_1) = f(a_2) \Rightarrow a_1 = a_2$ for every $a_1, a_2 \in A$.



Onto Function: A function $f : A \rightarrow B$ is one-one or surjective, if for each $b \in B$ there exists at least one $a \in A$ such that $f(a) = b$.



Composition of Functions: If $f : A \rightarrow B$ and $g : B \rightarrow C$ are two functions, then composition of functions $g \circ f : A \rightarrow C$ as $g \circ f(a) = g(f(a))$ for all $a \in A$.

INVERSE TRIGONOMETRIC FUNCTIONS

Principal Value Branches:

The value of inverse trigonometric functions which lies in its principal value branch is called the principal value of those inverse trigonometric functions.

Principal Value Branches:

Functions	Domain	Range (Principal Value Branches)
$y = \sin^{-1} x$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2} \right]$
$y = \cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$y = \operatorname{cosec}^{-1} x$	$\mathbf{R} - (-1, 1)$	$\left[-\frac{\pi}{2}, \frac{\pi}{2} \right] - \{0\}$
$y = \sec^{-1} x$	$\mathbf{R} - (-1, 1)$	$[0, \pi] - \left\{ \frac{\pi}{2} \right\}$
$y = \tan^{-1} x$	\mathbf{R}	$\left(-\frac{\pi}{2}, \frac{\pi}{2} \right)$
$y = \cot^{-1} x$	\mathbf{R}	$(0, \pi)$

Identities of Inverse Trigonometric Function:

(1)

$$(i) \sin^{-1} x = \operatorname{cosec}^{-1} \frac{1}{x}; x \geq 1 \text{ or } x \leq -1.$$

$$(ii) \cos^{-1} x = \sec^{-1} \frac{1}{x}; x \geq 1 \text{ or } x \leq -1 \text{ and}$$

$$(iii) \tan^{-1} x = \cot^{-1} \frac{1}{x}; x > 0.$$

(2)

$$(i) \sin^{-1}(-x) = -\sin^{-1} x; x \in [-1, 1]$$

$$(ii) \tan^{-1}(-x) = -\tan^{-1} x; x \in R$$

$$(iii) \operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x; |x| \geq 1$$

$$(iv) \cos^{-1}(-x) = \pi - \cos^{-1} x; x \in [-1, 1]$$

$$(v) \cot^{-1}(-x) = \pi - \cot^{-1} x; x \in R$$

$$(vi) \sec^{-1}(-x) = \pi - \sec^{-1} x; |x| \geq 1$$

(3)

$$(i) \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}; x \in [-1, 1],$$

$$(ii) \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}; x \in R \text{ and}$$

$$(iii) \sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}; |x| \geq 1.$$

(4)

$$(i) \tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}; xy < 1$$

$$(ii) \tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}; xy > -1$$

$$(iii) 2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}; |x| < 1$$

$$(iv) 2 \tan^{-1} x = \operatorname{siin}^{-1} \frac{2x}{1+x^2}; |x| \leq 1$$

Question: 1. If $f(x) = x + 7$ and $g(x) = x - 7$, $x \in \mathbb{R}$, find $(f \circ g)(7)$.

Question: 2. Solve: $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

Question: 3. (i) Is the binary operation $*$ defined on set \mathbb{N} , given by $a*b = \frac{a+b}{2}$ for all $a, b \in \mathbb{N}$ commutative?
(ii) Is the above binary operation $*$ associative?

Question: 4. Solve for x :

$$\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}.$$

Question: 5. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{2x-1}{3}$, $x \in \mathbb{R}$ is one-one and onto function. Also find the inverse of the function f .

Question: 6. Show that the Signum function of: $\mathbb{R} \rightarrow \mathbb{R}$ given by

$$f(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases} \text{ is neither one-one}$$

nor onto.

Question: 7. Show that the Relation R in the set $A = \{x \in \mathbb{Z} : 0 \leq x \leq 12\}$ is an equivalence relation. Where the relation R is given by

$$\{(a, b) : |a - b| \text{ is a multiple of } 4\}.$$

Question: 8. Solve the equation:

$$(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$$

Question: 9. Let $f: \mathbb{N} \rightarrow \mathbb{R}$ be a function defined as $f(x) = 4x^2 + 12x + 15$. Show that $f: \mathbb{N} \rightarrow S$ where, S is the range of f is invertible. Find the inverse of f .

Question: 10. Consider $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$ given by $f(1) = a$, $f(2) = b$ and $f(3) = c$. Find f^{-1} and show that $(f^{-1})^{-1} = f$.

MATRICES AND DETERMINANTS

- Let $A = [a_{ij}]_{m \times n}$ is a given matrix. Then the matrix obtained by interchanging the rows and columns is called the transpose of A. We write it as A' or A^T or $(A)^T$
- If $A = [a_{ij}]_{n \times n}$ is an $n \times n$ matrix such that $A^T = A$, then A is called symmetric matrix.

In a symmetric matrix, $a_{ij} = a_{ji}$

- If $A = [a_{ij}]_{n \times n}$ is an 'n x n' matrix such that $A^T = -A$, then A is called skew symmetric matrix. In a skew symmetric matrix, $a_{ij} = -a_{ji}$

If $i = j$, then $a_{ij} = -a_{ij} \Rightarrow a_{ij} = 0$

Inverse of a given matrix:

- Let A and B square matrices of order n such that $AB=BA=I$. Then A is called inverse of B and vice versa

We write $B=A^{-1}$ or $A=B^{-1}$

Minors and Co-factors of a Matrix:

Cofactor = $(-1)^{i+j} M_{ij}$

Properties of Determinants

Property 1:

If two rows or columns of a determinant are interchanged, then the sign of the determinant is changed.

Property 2

The value of a determinant remains unchanged if its rows and columns are interchanged.

Property 3

If each element of a row or column of a determinant is multiplied by a constant k, then its value gets multiplied by k.

Property 4

If in a determinant, the elements in two rows or columns are proportional, then the value of the determinant is zero.

Property 5

If the elements of a row (or column) of a determinant are expressed as sum of two terms, then the determinant can be expressed as sum of two determinants.

Property 6

If to any row or column of a determinant, a multiple of another row or column is added, the value of the determinant remains the same.

Formulae:

$$(i) \quad (A + B)^T = A^T + B^T$$

$$(kB)^T = kB^T, \text{ where } k \text{ is any constant}$$

$$(ii) \quad \Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$(iii) \quad A^{-1} = \frac{1}{|A|} \text{adj}A$$

Question1: Using matrix method, solve the following equations:

$$2x + 6y = 2$$

$$3x - z = -8$$

$$2x - y + z = -3.$$

Question: 2. If $A = \begin{pmatrix} 2 & -3 \\ 3 & 4 \end{pmatrix}$, show that $A^2 - 6A + 17I = 0$. Hence find A^{-1}

Question: 3. Obtain the inverse of the following matrix using elementary operations.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}.$$

Question: 4. Using properties of determinants, prove the following:

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

Question: 5. Express the following matrix as the sum of a symmetric and a skew symmetric matrix.

$$\begin{bmatrix} 1 & 3 & 5 \\ -6 & 8 & 3 \\ -4 & 6 & 5 \end{bmatrix}.$$

Question: 6. If $A = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ then prove that $A^n = \begin{bmatrix} \cos n\theta & \sin n\theta \\ -\sin n\theta & \cos n\theta \end{bmatrix}, n \in N$

Question: 7. If a, b, c are A.P then show that:
$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix} = 0$$

Question: 8. If A and B are two matrices such that $AB = B$ and $BA = A$, then Find $A^2 + B^2$.

CONTINUITY AND DIFFERENTIABILITY

Continuity of a Function at a Point: A real function $f(x)$ is said to be continuous at a point 'a' if limit of $f(x)$ at x is equal to $f(a)$.

$\therefore f(x)$ is continuous at $x = a$, if

$$\lim_{x \rightarrow a^+} f(x) = \lim_{x \rightarrow a^-} f(x) = f(a)$$

If a function is not continuous at point a , then the function is discontinuous and 'a' is the **point of discontinuity**.

Continuity of a Function in an Interval: A real function is continuous in an interval, if the function is continuous at every point of the interval.

A function $f(x)$ is continuous in a closed interval $[a, b]$,

if $\lim_{x \rightarrow a^+} f(x) = f(a)$ and $\lim_{x \rightarrow b^-} f(x) = f(b)$.

If f and g are two continuous functions, then $f + g$, $f - g$, fg and cf are also continuous. The function f/g is also continuous when g is a non-zero function.

Some Results on Continuity:

1. Every constant function is continuous.
2. Every polynomial function is continuous.
3. The exponential function is continuous.
4. The logarithmic function $\log_e x$ is continuous.
5. $\sin x$ and $\sin^{-1}x$ are continuous functions.

Differentiability: A $f(x)$ is a real function and a is a real number, then

$Rf'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ is right hand derivative and $Lf'(a) = \lim_{h \rightarrow 0} \frac{f(a-h) - f(a)}{-h}$ is left hand derivative. If $Rf'(a) = Lf'(a)$, then $f(x)$ is differentiable at $x = a$.

Remarks: A differentiable function is continuous, but a continuous function need not be differentiable.

Derivative of Some Basic Functions:

- (1) $\frac{d}{dx}(C) = 0$; Where C is a constant
- (2) $\frac{d}{dx} x^n = nx^{n-1}$
- (3) $\frac{d}{dx} e^x = e^x$
- (4) $\frac{d}{dx} a^x = a^x \log a$
- (5) $\frac{d}{dx} (\log x) = \frac{1}{x}$
- (6) $\frac{d}{dx} \sin x = \cos x$
- (7) $\frac{d}{dx} \cos x = -\sin x$
- (8) $\frac{d}{dx} (\tan x) = \sec^2 x$
- (9) $\frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x$
- (10) $\frac{d}{dx} (\sec x) = \sec x \tan x$
- (11) $\frac{d}{dx} (\operatorname{cosec} x) = -\operatorname{cosec} x \cot x$

Question 1: Find the value of k, if the function.

$$f(x) = \begin{cases} kx^2, & x \geq 1 \\ 4, & x < 1 \end{cases} \text{ is continuous at } x = 1$$

Question: 2. Find all the points of discontinuity of the function f defined by

$$f(x) = \begin{cases} x+2, & x \leq 1 \\ x-2, & 1 < x < 2 \\ 0, & x \geq 2 \end{cases}$$